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Polynomials: terminology and introduction

A polynomial in one variable is an expression of the form

$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0$$

Where $x \in \mathbb{R}$, every exponent is a positive integer, and the coefficients are complex numbers. If the highest power of x is x^k, then the polynomial is said to be of degree k.

In our work, the coefficients will be restricted to the rational numbers.

A polynomial function has the form

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0$$

A polynomial equation has the form

$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0 = 0$$

The **zeros** of a polynomial are those values of the independent variable for which the polynomial takes the value zero; i.e. all x such that $P(x) = 0$. For example, the numbers 2 and 3 are zeros of $P(x) = x^2 - 5x + 6$.

The **roots** of a polynomial equation, also known as the **solutions**, are the values of the unknown that make the equation true. For example, the numbers 2 and 3 are roots of the equation $x^2 - 5x + 6 = 0$.

When the context makes clear what is intended, one often says "polynomial" instead of the longer phrases mentioned above. The words "zeros", "roots", and "solutions" are used similarly loosely.

A "higher degree polynomial" usually means a polynomial of degree 3 or greater.

Polynomials of some degrees have special names:

name	degree
quadratic	2
cubic	3
quartic	4
quintic	5

The Fundamental Theorem of Algebra essentially asserts that a polynomial of degree k has k roots in the complex numbers, though roots may be of multiplicity greater than 1. Thus, $x^3 = 1$ and $x^7 = 1$ have 3 and 7 roots respectively.

An easy formula exists to find the roots of any quadratic equation. Formulae exist to find the roots of cubic and quartic equations, but the formulae are very complex. Niels Henrik Abel (1802-1829) proved that no formulae exist that will solve general polynomial equations of degree 5 or higher.

When an exact solution is not required, for example in engineering, sophisticated techniques of calculus may be used to find approximate solutions of higher degree polynomials.

In our work, we will find exact solutions to polynomials that are easily solved.